中国科学院大学
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Generative Tensor Network Classification for Supervised Learning

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Sun Z Z, Peng C, Liu D, et al. Generative tensor network classification model for supervised machine learning[J]. Physical Review B, 2020, 101(7): 075135.

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1 Background: Classifying images in quantum space

Step one: Mapping images to the many-body Hilbert space

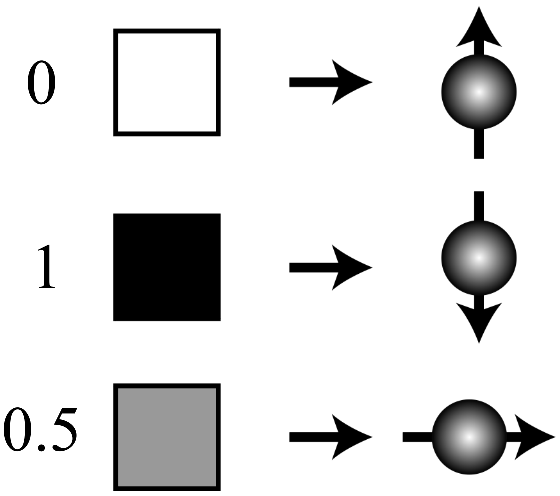
Step two: Classifying images by distance

Mapping images to the many-body Hilbert space

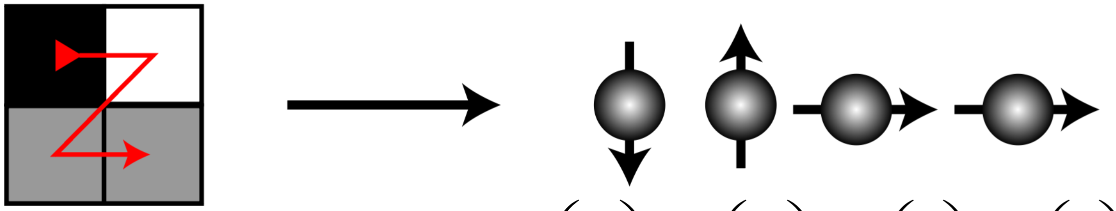
pixel: $x(x\in[0,1])$

$$\downarrow$$

$$\begin{pmatrix}\cos(x\pi/2)\\ \sin(x\pi/2)\end{pmatrix}$$



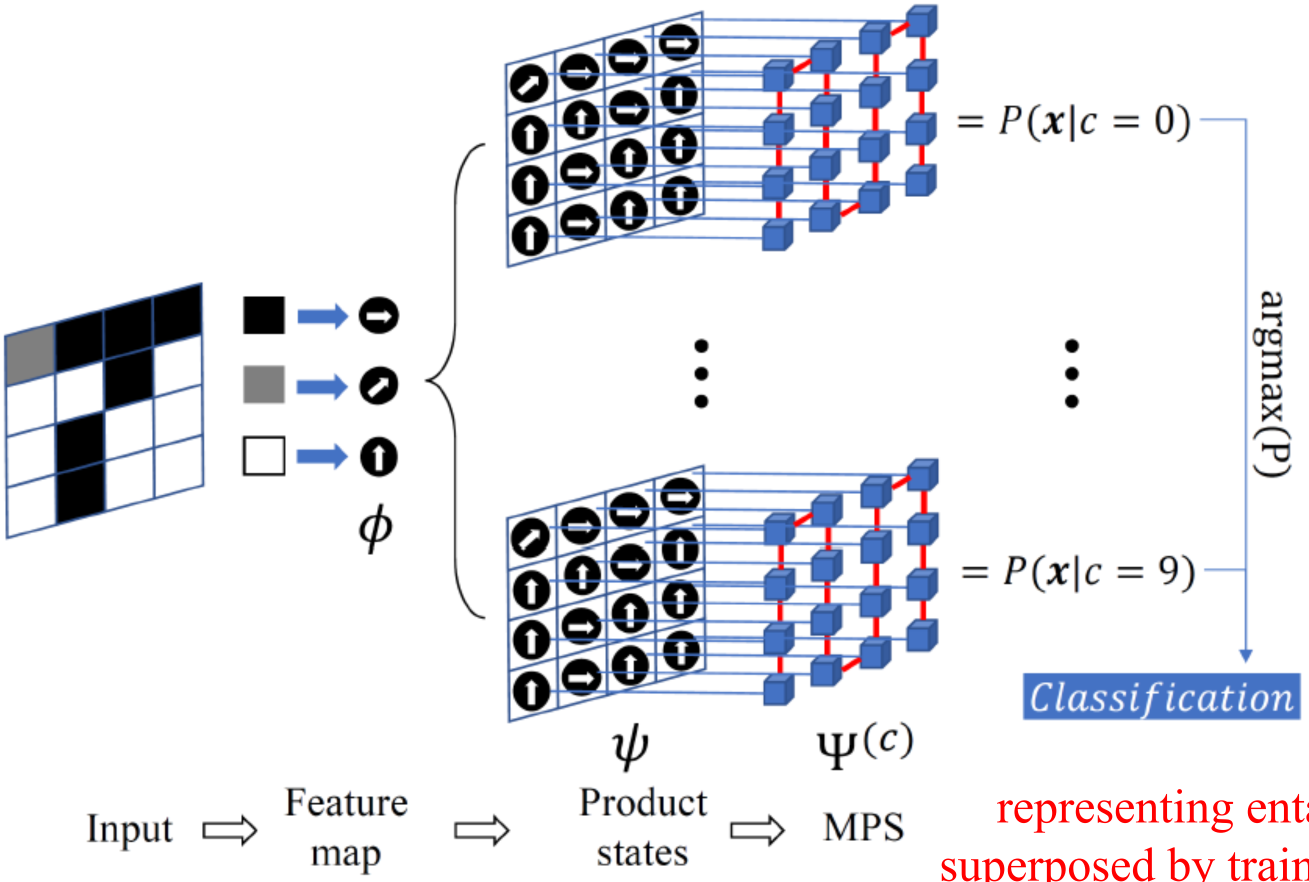
$$0 \rightarrow \begin{pmatrix}1\\0\end{pmatrix} \qquad 1 \rightarrow \begin{pmatrix}0\\1\end{pmatrix} \qquad 0.5 \rightarrow \frac{\sqrt{2}}{2}\begin{pmatrix}1\\1\end{pmatrix}$$



$$\begin{pmatrix}1 & 0 & 0.5 & 0.5\end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\1\end{pmatrix}\otimes\begin{pmatrix}1\\1\end{pmatrix}$$

Classifying images by distance (fidelity)



The probability is defined as the square of fidelity:

$P(x|c=9) = \langle \psi | \Psi^{(9)} \rangle^2$

representing entangled state superposed by training set images

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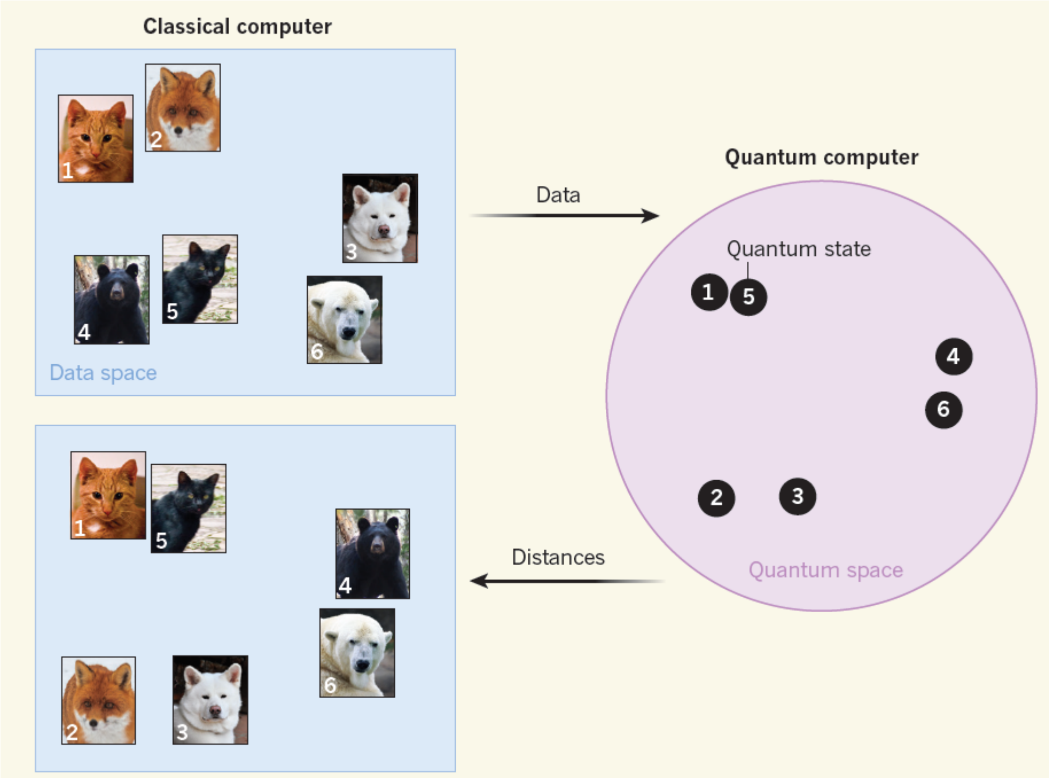
How are images distributed in quantum space

◎Why bother to map images to quantum space

◎Distribution of images in different spaces in MNIST dataset

Why bother to map images to quantum space?



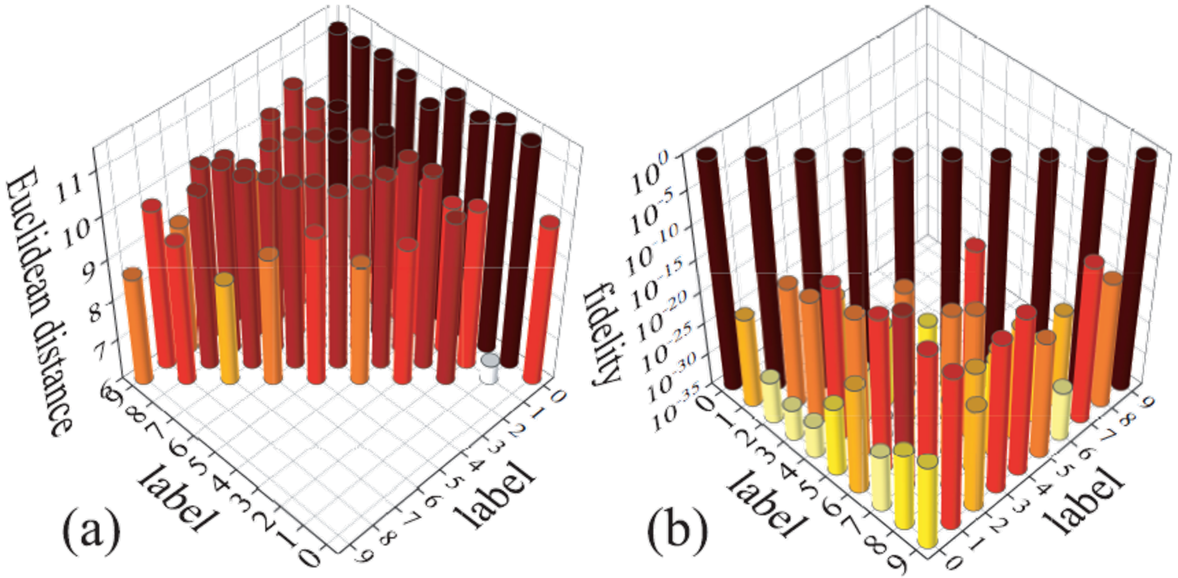
Is this true?

We hope images with the same label are closer.

Schuld M. Machine learning in quantum spaces[J]. Nature, 2019, 567(7747): 179.

Distribution of images in different spaces

Average Euclidean distances (a) and fidelities (b) between the samples of MNIST in the original (a) and quantum space (b)[1].



$$D_{c_1 c_2} = \frac{1}{N_{c_1} N_{c_2}} \sum_{\mathbf{x}\in c_1} \sum_{\mathbf{y}\in c_2} \left|\mathbf{x}-\mathbf{y}\right|$$

$$F_{c_1 c_2} = \frac{1}{N_{c_1} N_{c_2}} \sum\nolimits_{\mathbf{u}\in c_1,\mathbf{v}\in c_2,\mathbf{u}\neq\mathbf{v}} \left|\mathbf{u}^{\dagger}\mathbf{v}\right|$$

3 Performance of generative tensor network classification

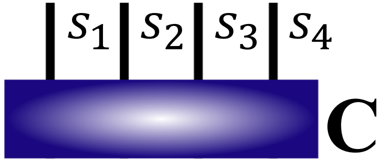
◎Tensor network representation for an entangled state

◎Comparison of testing accuracy

Tensor network representation for an entangled state

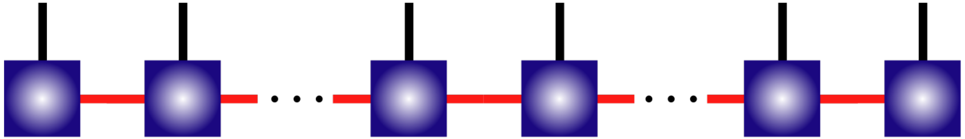
Entangled state: $$\left|\Psi\right\rangle = \sum_{s_1 s_2 \cdots s_N} \mathbf{C}_{s_1 s_2 \cdots s_N} \prod_{n}^{N} \left|s_n\right\rangle$$

Tensor representation:

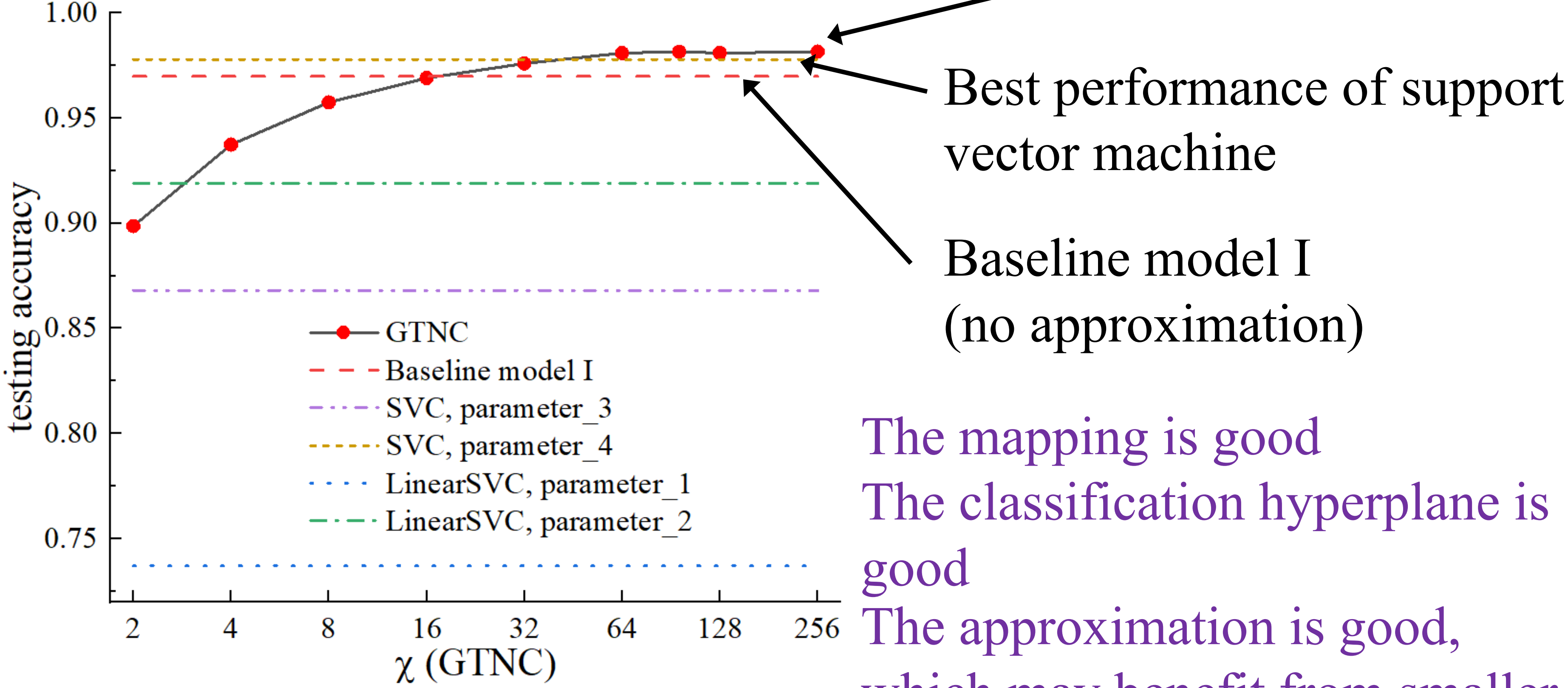


Approximately

Matrix product state representation:



Comparison of testing accuracy



The mapping is good

The classification hyperplane is good

The approximation is good, which may benefit from smaller entanglement entropy

4 Contributions

◎Images with the same label are closer in quantum space

◎Generative tensor network can obtain a good classification hyperplane in quantum space

THANKS

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