

Bayesian Tensor Ring Decomposition for Low Rank Tensor Completion

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January 8th, 2021

Outline

Introduction

Model Formulation

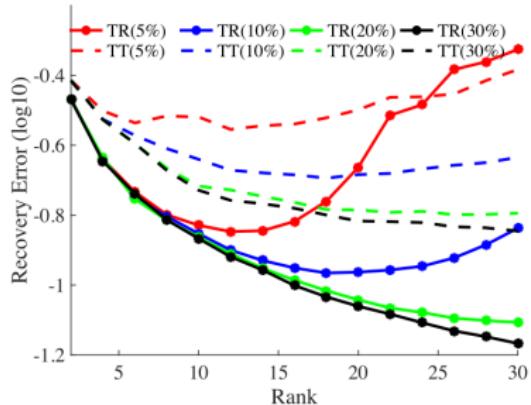
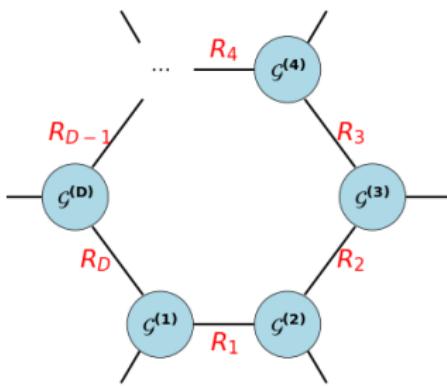
Numerical Results

Tensor Ring Completion

(Wang, et al., ICCV 2017)

For a partially observed tensor \mathcal{Y} , the tensor ring completion problem is,

$$\hat{\mathcal{X}} = \arg \min_{\mathcal{X}} \|\mathcal{Y} - \mathcal{X}\|_{\Omega}^2, \quad s.t., \quad \mathcal{X} = \ll \mathcal{G}^{(1)}, \dots, \mathcal{G}^{(D)} \gg.$$



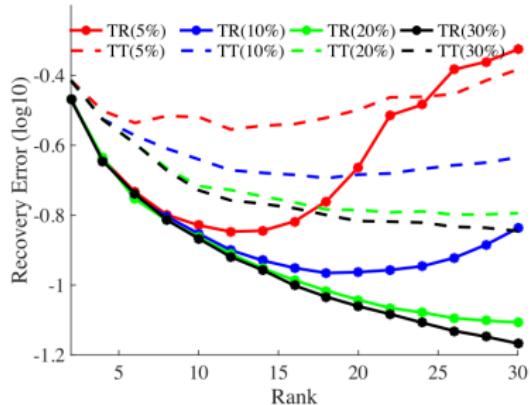
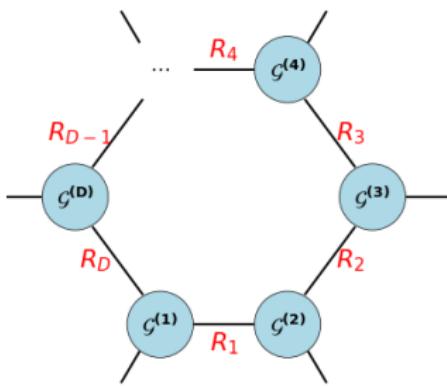
How to choose the ranks R ?

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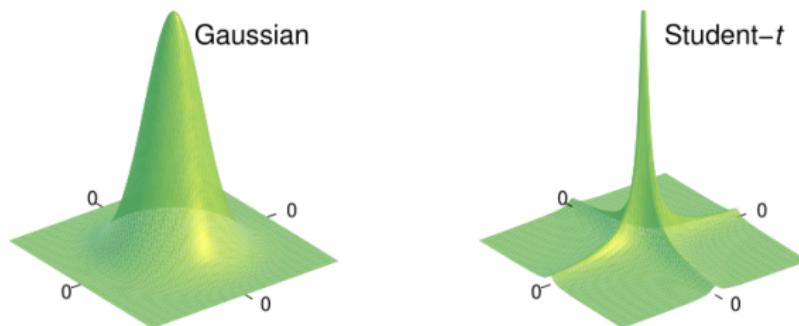
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Sparse Bayesian Learning

(Tipping, JMLR 2001)

The automatic relevance determination (ARD) prior,

$$p(\mathbf{G} | \mathbf{u}) = \mathcal{N}(\mathbf{0}, \text{diag}(1/\mathbf{u}^{-1})), \quad u_i \sim \Gamma(a_0^u, b_0^u).$$



Related Work

Bayesian CP Decomposition (Zhao, et al., TPAMI 2015)

Automatically select CP rank, but is less expressive than TR.

Tensor Ring Completion (Wang, et al., ICCV 2017; Yuan, et al., AAAI 2019)

Empirically select ranks or add regularization terms (still needs hyper-parameters).

Unable to infer the true rank.

Bayesian Tensor Ring Decomposition

Using the ARD prior, the Bayesian tensor ring decomposition (BTRD) is

$$\mathbf{y}_\Omega | \{\mathbf{G}^{(i)}\}_i, \tau \sim \prod_{i_1=1}^{l_1} \cdots \prod_{i_D=1}^{l_D} \mathcal{N}(y_i | \text{tr}(\mathbf{G}^{(1)}[i_1] \cdots \mathbf{G}^{(D)}[i_D]), \tau^{-1})^{\mathcal{O}_i},$$

$$\tau \sim \Gamma(c_0, d_0),$$

$$\mathbf{G}^{(d)}[i_d] | \mathbf{U}^{(d)}, \mathbf{U}^{(d+1)} \sim \mathcal{MN}(0, \mathbf{U}^{(d)}, \mathbf{U}^{(d+1)}), \quad \mathbf{u}_i^{(d)} \sim \Gamma(a_0, b_0),$$

for $d = 1, \dots, D$.

Prune small factors during training.

Variational Inference

If we use a *factorized* variational posterior, since the model is conjugate, the update rule is,

$$\ln q_j^*(\Theta_j) = \langle \ln p(\mathcal{Y}_\Omega, \Theta) \rangle_{q(\Theta \setminus \Theta_j)} + \text{const.}$$

Compute the expectations by **tensor contraction** diagram,

$$\langle (\mathbf{G}^{\neq 5}[\bar{\mathbf{i}}_{-5}])^\top \circ (\mathbf{G}^{\neq 5}[\bar{\mathbf{i}}_{-5}])^\top \rangle = \begin{array}{c} \text{Diagram showing tensor contraction between two layers of tensors } \langle \mathcal{A}^{(d)} \rangle \text{ and } \langle \mathcal{A}^{(d+1)} \rangle. \\ \text{The diagram consists of two rows of four boxes each. The top row contains boxes labeled } \langle \mathcal{A}^{(1)} \rangle, \langle \mathcal{A}^{(2)} \rangle, \langle \mathcal{A}^{(3)} \rangle, \text{ and } \langle \mathcal{A}^{(4)} \rangle \text{ from left to right. The bottom row contains boxes labeled } \langle \mathcal{A}^{(2)} \rangle, \langle \mathcal{A}^{(3)} \rangle, \langle \mathcal{A}^{(4)} \rangle, \text{ and } \langle \mathcal{A}^{(1)} \rangle \text{ from left to right.} \\ \text{Connections between boxes in adjacent columns are labeled with indices: } I_1, R_1, R_{I_1}, R_{I_2}, R_{I_3}, R_{I_4}, R_2, R_3, R_4, R_5. \\ \text{Indices } I_1, I_2, I_3, I_4 \text{ are shown above their respective columns, while } R_1, R_2, R_3, R_4, R_5 \text{ are shown below them.} \end{array},$$

where

$$\mathcal{A}^{(d)} = \mathcal{G}^{(d)} \circ \mathcal{G}^{(d)} \in \mathbb{R}^{I_d \times R_d \times R_{(d+1)} \times R_d \times R_{(d+1)}}.$$

Simulation Study

Infer the **true rank** of synthetic data.

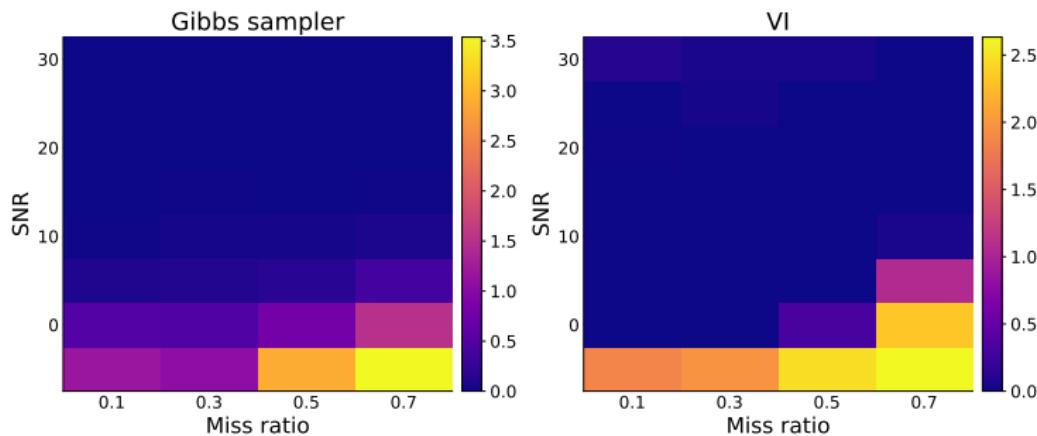


Figure 1: Rank estimation error.

Image Inpainting

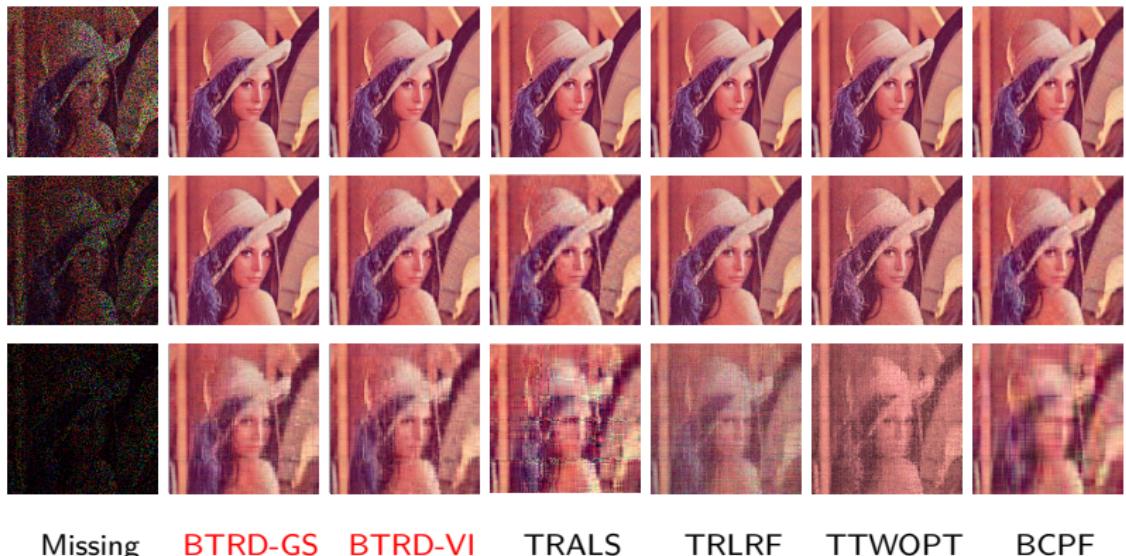


Figure 2: Image inpainting for missing ratio 0.5, 0.7, 0.9.

Thanks for your listening!