



Acceleration of Fractional Fourier Transform via Tensor- train Decomposition

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The presentation discusses:

- Tensor-train enabled method to accelerate the numerical calculation of discrete fractional Fourier transform (FrFT)
- FrFT's potential use in optics, signal processing and differential equations.



Define the a th order FrFT using the following linear integral transform:

$$f_a(u) = \int K_a(u, v) f(v) dv,$$

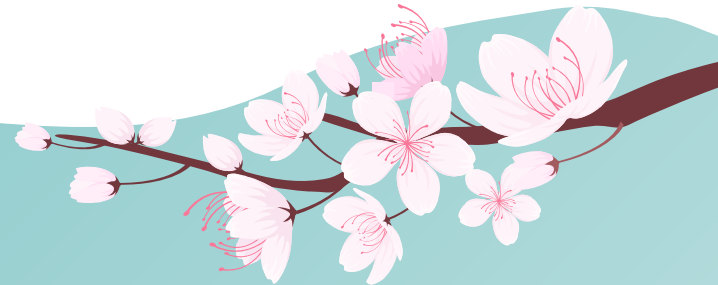
where

$$K_a(u, v) = A_\phi \exp[\imath\pi(\cot\phi u^2) - 2\csc\phi uv + \cot\phi v^2],$$

and


$$\phi = \frac{a\pi}{2}$$

$$A_\phi = \sqrt{1 - \imath \cot\phi}$$



The discrete form of FrFT is given as

$$\mathcal{F}^{\alpha} f\left(\frac{m}{2\Delta x}\right) = \frac{A_{\phi}}{2\Delta x} \sum_{n=-N}^N e^{i\pi\left(\alpha\left(\frac{m}{2\Delta x}\right)^2 - 2\beta\frac{m*n}{(2\Delta x)^2}\right) + \alpha\left(\frac{n}{2\Delta x}\right)^2} f\left(\frac{n}{2\Delta x}\right)$$



Haldun M Ozaktas, Orhan Arikan, M Alper Kutay, and Gozde Bozdogan. Digital computation of the fractional fourier transform. *IEEE Transactions on signal processing*, 44(9):2141–2150, 1996.

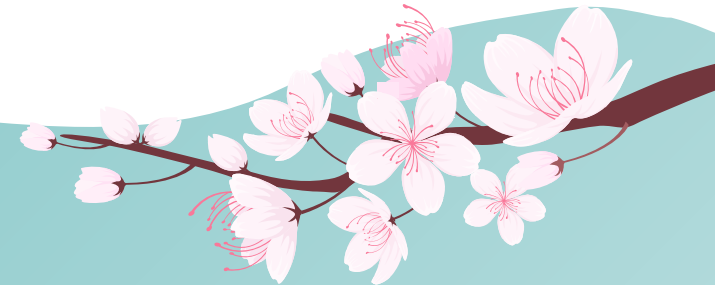
Apply Tensor Train to further acceleration the computation of FrFT:

$$\mathbf{f}_a = (\mathbf{D}\Lambda\mathbf{H}_{l_p}\Lambda\mathbf{J}) \mathbf{f},$$

where $\mathbf{f}, \mathbf{f}_a \in \mathcal{C}^N$ denotes the input and output vector over the complex linear space, respectively. In Eq. (6), \mathbf{D} and

$$\underline{\mathbf{f}} \otimes \underline{\mathbf{v}} = \left(\underline{\mathbf{f}}^{(1)} \odot \underline{\mathbf{v}}^{(1)} \right) \times^1 \left(\underline{\mathbf{f}}^{(2)} \odot \underline{\mathbf{v}}^{(2)} \right) \times^1 \dots \times^1 \left(\underline{\mathbf{f}}^{(p)} \odot \underline{\mathbf{v}}^{(p)} \right)$$

where \otimes, \odot and \times^1 denote the Hadamard product, partial Kronecker product and tensor contraction of two tensors, respectively. $\underline{\mathbf{f}}^{(i)}$ and $\underline{\mathbf{v}}^{(i)}$, $i \in [p]$ denotes the core tensors $\underline{\mathbf{f}}$ and $\underline{\mathbf{v}}$, respectively.





Thank You