Acceleration of Fractional Fourier Transform via Tensor-train Decomposition

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The presentation discusses:

- Tensor-train enabled method to accelerate the numerical calculation of of discrete fractional Fourier transform (FrFT)
- FrFT’s potential use in optics, signal processing and differential equations.

Define the ath order FrFT using the following linear integral transform:

$$f_a(u) = \int K_a(u, v) f(v) dv,$$

where

$$K_a(u, v) = A_\phi exp[\iota\pi(cot\phi u^2) - 2csc\phi uv + cot\phi v^2],$$

and

$$\phi = \frac{a\pi}{2}$$

$$A_\phi = \sqrt{1 - \iota cot\phi}$$

The discrete form of FrFT is given as

$$\mathcal{F}^a f\left(\frac{m}{2\Delta x}\right) = \frac{A_\phi}{2\Delta x} \sum_{n=-N}^{N} e^{\iota\pi(\alpha(\frac{m}{2\Delta x}^2 - 2\beta\frac{m*n}{(2\delta x)^2})) + \alpha(\frac{n}{2\Delta x})^2} f\left(\frac{n}{2\Delta x}\right).$$

Haldun M Ozaktas, Orhan Arikan, M Alper Kutay, and Gozde Bozdagt. Digital computation of the fractional fourier transform. *IEEE Transactions on signal processing*, 44(9):2141–2150, 1996.

Apply Tensor Train to further acceleration the computation of FrFT:

$$\mathbf{f}_a = \left(\mathbf{D}\mathbf{\Lambda}\mathbf{H}_{l_p}\mathbf{\Lambda}\mathbf{J}\right)\mathbf{f},$$

where $\mathbf{f}$, $\mathbf{f}_a \in \mathcal{C}^N$ denotes the input and output vector over the complex linear space, respectively. In Eq. (6), $\mathbf{D}$ and

$$\underline{\mathbf{f}} \otimes \underline{\mathbf{v}} = \left(\underline{\mathbf{f}}^{(1)} \odot \underline{\mathbf{v}}^{(1)}\right) \times^1 \left(\underline{\mathbf{f}}^{(2)} \odot \underline{\mathbf{v}}^{(2)}\right) \times^1 \cdots \times^1 \left(\underline{\mathbf{f}}^{(p)} \odot \underline{\mathbf{v}}^{(p)}\right)$$

where $\otimes$,$\odot$ and $\times^1$ denote the Hadamard product, partial Kronecker product and tensor contraction of two tensors, respectively. $\underline{\mathbf{f}}^{(i)}$ and $\underline{\mathbf{v}}^{(i)}$, $i \in [p]$ denotes the core tensors $\underline{\mathbf{f}}$ and $\underline{\mathbf{v}}$, respectively.

Thank You