DTAE: Deep Tensor Autoencoder for 3-D Seismic Data Interpolation

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Introduction

The core challenge of seismic data interpolation is how to capture latent spatial-temporal relationships between unknown and known traces in 3-D space. This article presents a basic deep tensor autoencoder (DTAE) and two variants to implicitly learn a data driven, nonlinear, and high-dimensional mapping to explore the complicated relationship among traces without the need for any underlying assumption. The performance benefits of the proposed DTAE-based method are demonstrated in experiments with both synthetic and real field seismic data.

Problem Statement

Problem statement: If we assume that the missing traces are randomly distributed, the observation or sampling process can be represented as

$$\mathcal{T}_\Omega = \mathcal{P}_\Omega(\mathcal{T}). \tag{1}$$

Here, $\mathcal{P}_\Omega(\cdot)$ denotes the sampling operator, which projects $\mathcal{T}$ onto the set Ω according to the following:

$$\mathcal{P}_\Omega(\mathcal{T}) = \begin{cases} \mathcal{T}(i,j,:), & \text{if } (i,j) \in \Omega, \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where the (i,j)-th vector of $\mathcal{P}_\Omega(\mathcal{T})$ is equal to $\mathcal{T}(i,j,:)$ if $(i,j) \in \Omega$ and zero otherwise.

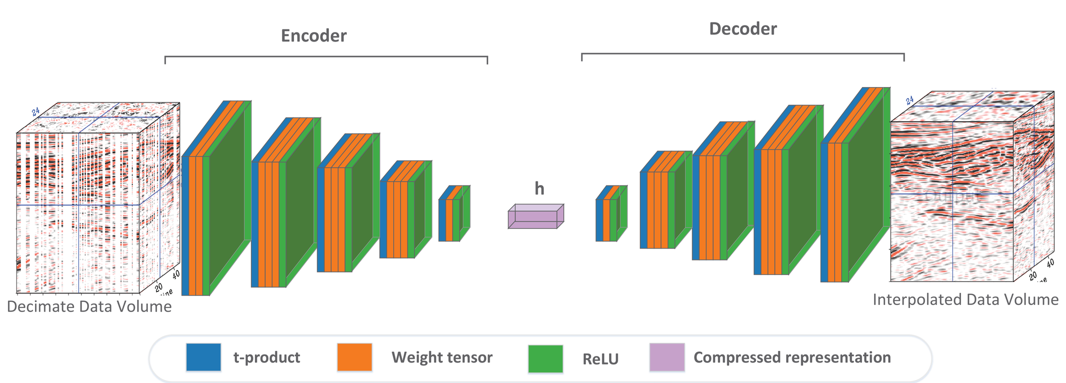
Problem Formulation: By introducing DL regression $f(\mathcal{T},\Theta)$, the learning objective of the interpolation task becomes

$$\mathcal{L}_1(\mathcal{T},\mathcal{T}_\Omega;\Theta) = \underset{\Theta}{\operatorname{argmin}}\ \mathbb{E}_{\mathcal{T}_\Omega} \|f(\mathcal{T},\Theta) - \mathcal{T}_\Omega\|_F^2 \tag{3}$$

From this formulation, it is clear that the task here is to learn a function $f(\cdot)$ with respect to Θ that best approximates $\eta^{-1}(\cdot)$. Our final solution, $\widehat{\mathcal{T}}$, can be obtained as follows:

$$\widehat{\mathcal{T}} = \eta(\mathcal{T}_\Omega) \odot (1 - \Omega) + \mathcal{T}_\Omega \tag{4}$$

DTAE model



Network architecture: Given a pair of training samples $(\mathcal{X}_n, \mathcal{Y}_n)$, we can construct a new DTAE model with the structure. Similar to the matrix AE presented, DTAE is also a t-NN with encoding and decoding layers.

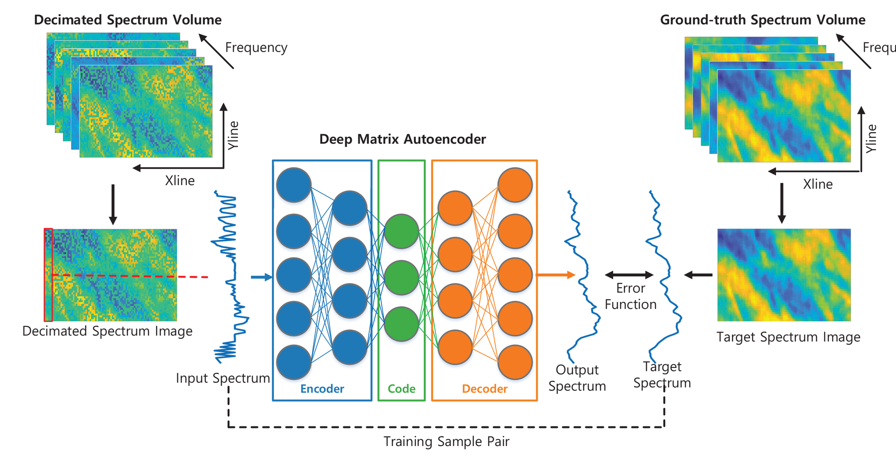
$$\mathcal{X}_{n,\ell} = \mathbf{h}(\mathcal{X}_{n,\ell-1}) = \sigma\left(\mathcal{W}_{\ell-1} * \mathcal{X}_{n,\ell-1} + \vec{\mathcal{B}}_\ell\right) \tag{5}$$

$$\mathbf{g}(\mathcal{X}_{n,\ell-1}) = \sigma\left(\mathcal{W}_{\ell-1}^\dagger * \mathbf{h}(\mathcal{X}_{n,\ell-1}) + \vec{\mathcal{B}}_{\ell-1}\right) \tag{6}$$

Objective function: In our supervised learning process, training the DTAE model involves finding the parameters $\Theta = \{\mathcal{W}_\ell, \mathcal{B}_\ell\}_{\ell=1}^K$ such that the expected interpolation error between the output tensor values $\{\mathcal{X}_{n,\ell}\}_{n=1}^N$ and the desired tensor values $\{\mathcal{Y}_n\}_{n=1}^N$ is minimized:

$$\mathcal{L}_2(\mathcal{X}_n,\mathcal{Y}_n;\Theta) = \arg\min_\Theta \frac{1}{N}\sum_{n=1}^N \frac{1}{2}\|\mathcal{X}_{n,\ell} - \mathcal{Y}_n\|_F^2 \tag{7}$$

TBP Algorithm



TBP algorithm: To optimize model (8), we develop a TBP algorithm to minimize $\mathcal{L}_3(\mathcal{X}_n,\mathcal{Y}_n;\Theta)$ as a function of Θ. That is, we adjust the network parameters Θ through layer evolution rules. First, we need to determine how gradient descent modifies $\mathcal{W}_{\ell-1}$ and $\mathcal{B}_\ell$:

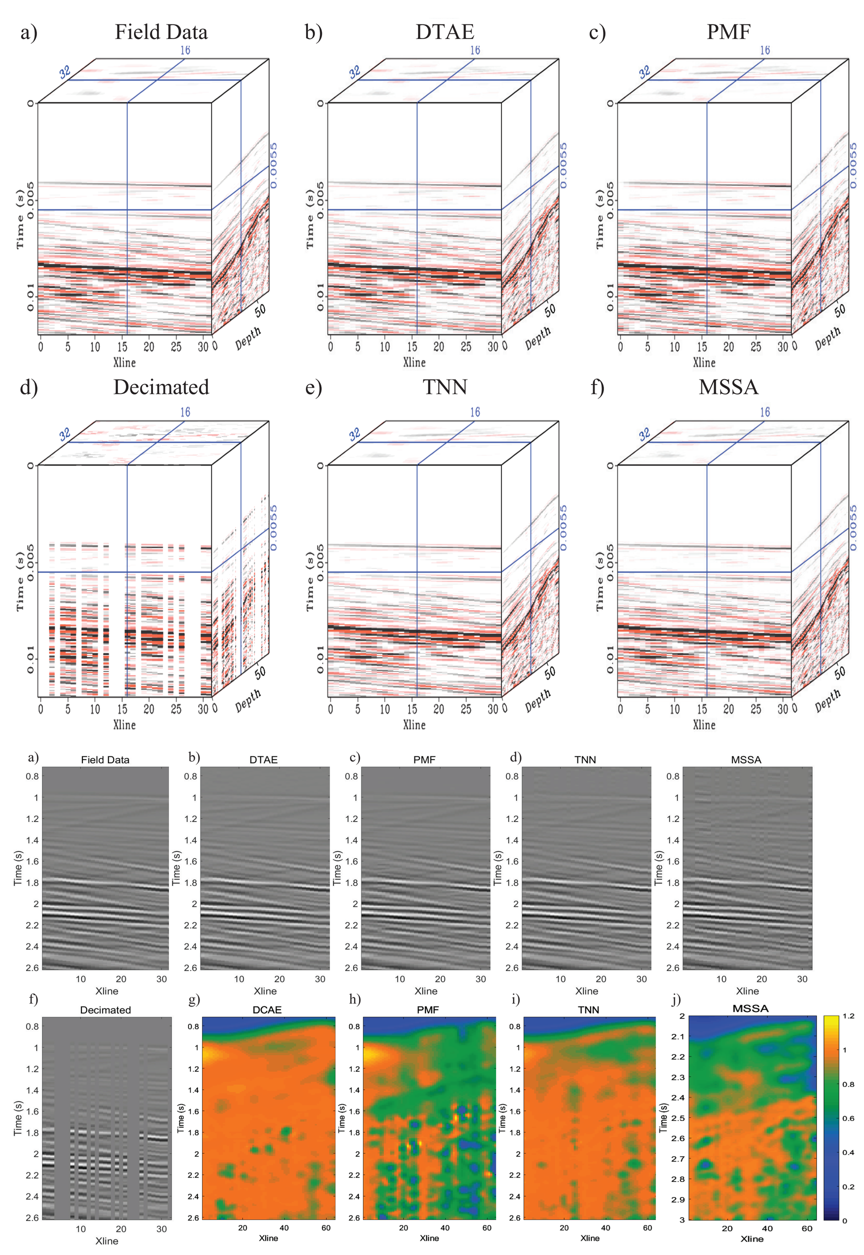
$$\begin{aligned} \mathcal{W}_{\ell-1} &\leftarrow \mathcal{W}_{\ell-1} - \alpha\frac{\partial\mathcal{L}_3(\mathcal{W},\mathcal{B})}{\partial\mathcal{W}_{\ell-1}} \\ \mathcal{B}_\ell &\leftarrow \mathcal{B}_\ell - \alpha\frac{\partial\mathcal{L}_3(\mathcal{W},\mathcal{B})}{\partial\mathcal{B}_\ell} \end{aligned} \tag{8}$$

Implementation details: Taking advantage of this relationship, we derive a solid theoretical framework in which the whole DTAE solution is split into an individual deep matrix autoencoder (DMAE) solution for each frontal slice in the discrete cosine transform (DCT) domain.

Algorithm 1 TBP Algorithm
Require: $\mathcal{X}_n, \mathcal{Y}_n, \alpha, N_{\text{bs}}, N_r, N_t, \mathcal{P}_\Omega, \mathbb{P}_{AE}$, Type.
Ensure: $\widehat{\mathcal{X}}_n$.
// Training of the DTAE model
$\widetilde{\mathcal{X}}_n \leftarrow \text{dct}(\mathcal{X}_n, [\,], 3)$, $\widetilde{\mathcal{Y}}_n \leftarrow \text{dct}(\mathcal{Y}_n, [\,], 3)$.
for $k = 1, \cdots, N_3$ **do**
  $f_\theta(\cdot) \leftarrow \text{DMAE}(\widetilde{\mathcal{X}}_n^{(k)},\ \widetilde{\mathcal{Y}}_n^{(k)},\ \alpha,\ N_{\text{bs}},\ \mathbb{P}_{AE},\ \text{type})$.
end for
// Testing of the DTAE model
for $k = 1, \cdots, N_3$ **do**
  $\widetilde{\mathcal{P}}_n^{(k)} \leftarrow f_\theta(\widetilde{\mathcal{X}}_n^{(k)}),\ n \in [N_t]$.
end for
$\widehat{\mathcal{X}}_n \leftarrow \text{idct}(\widetilde{\mathcal{P}}_n, [\,], 3),\ n \in [N_t]$.
$\widehat{\mathcal{X}}_n = \eta(\widehat{\mathcal{X}}_n) \odot (1 - \mathcal{P}_\Omega) + \widehat{\mathcal{X}}_n$.

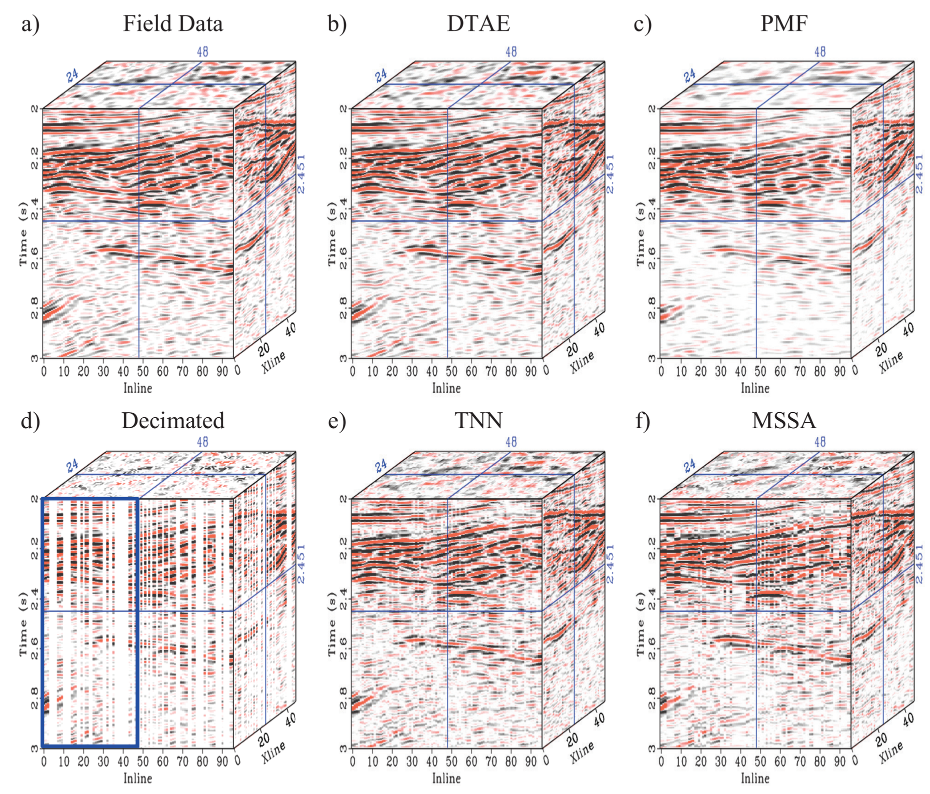
Validation on Synthetic Data

Experiment result: For synthetic VSP data, the superior performance of the DTAE model is clearly apparent.

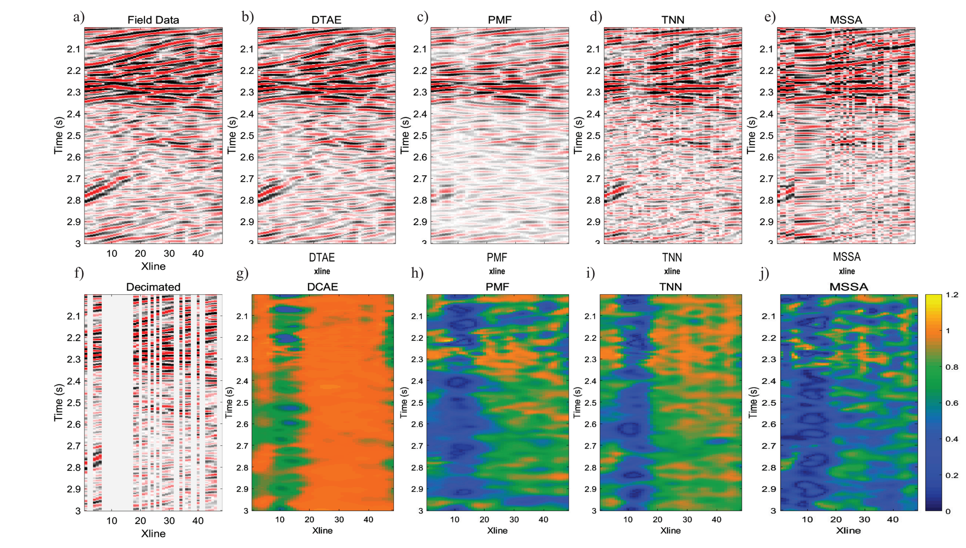


Validation on Parihaka-3D Data

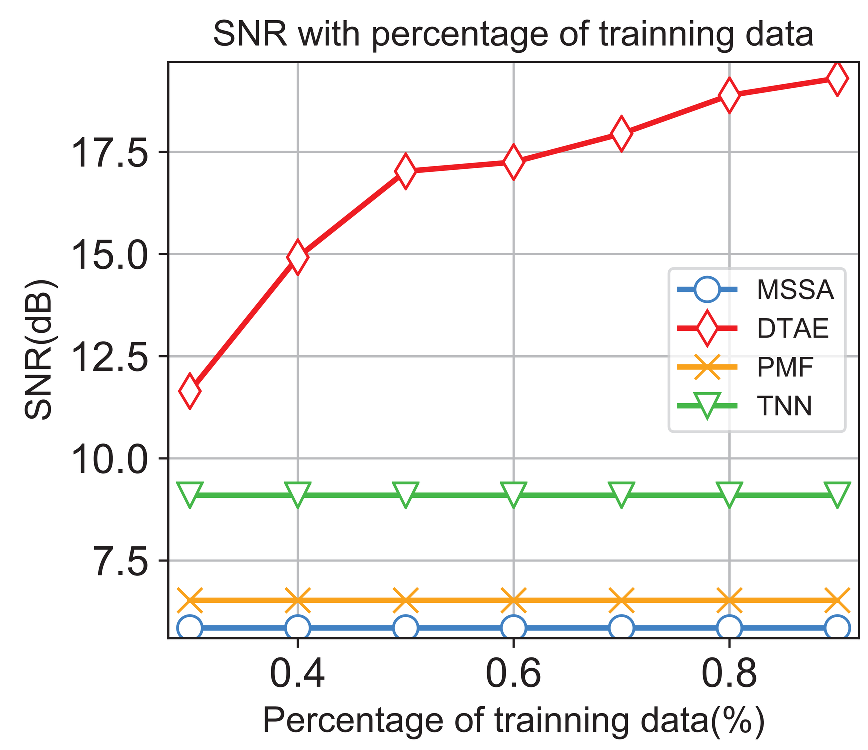
Experiment result: To further verify the flexibility of the DTAE method, we apply our method to the Parihaka-3D seismic survey recorded from complex geological structures, with the goal of demonstrating the substantial performance gains of DTAE in the case that the low-rank assumption is not satisfied.



Note that the DTAE network has a nice performance on reconstruction of big gap.



SNR comparison with respect to different percentages of training data in the Parihaka-3D experiment:



Conclusions

In this article, we proposed a basic DTAE model and two variants for learning a data-driven, nonlinear, high-dimensional mapping to discover the complicated relationship among seismic traces. Experimental results obtained on a synthetic dataset and two real field datasets demonstrated that the proposed DTAE model can achieve smaller interpolation errors than three SOTA interpolation methods.

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References
[1] F. Qian, Z. Liu, Y. Wang, S. Liao, S. Pan, and G. Hu, "Dtae: Deep tensor autoencoder for 3-d seismic data interpolation," IEEE Trans. Geosci. Remote Sens., early access, May 6, 2021, doi: 10.1109/TGRS.2021.3075968.