

# Multi-dimensional Visual Data Completion via Low-rank Tensor Representation under Coupled Transform

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## Background: Tensor Decomposition

Tensor decomposition aims to decompose a higher-order tensor to a set of low-dimensional factors and has powerful capability to capture the global correlations of data.

CANDECOMP/PARAFAC (CP) decomposition:

$$\mathcal{X} = \sum_{r=1}^R \lambda_r a_r \circ b_r \circ c_r,$$

Tucker decomposition:

$$\mathcal{X} = \mathcal{G} \times_1 A \times_2 B \times_3 C,$$

Tensor singular values decomposition:

$$\mathcal{X} = U * \mathcal{S} * V^H,$$

They only consider the global data correlation (i.e., low-rankness) but ignore the spatial multi-scale nature.

## Contributions

We propose an enhanced low-rank tensor representation (LRTR) under coupled transform, which provides a novel perspective to exploit the implicit low-rank structure.

We propose the CT-LRTC model by the enhanced LRTR for multi-dimensional visual data restoration.

Extensive real examples on color images, multispectral images, and videos illustrate the proposed method outperforms many state-of-the-art methods in qualitative and quantitative aspects.

## Enhanced Low-Rank Tensor Representation

The enhanced LRTR under coupled transform aims to explore suitable transform to decorrelate the spatial and temporal/spectral dimensions, achieving a better low-rank approximation.

In the first layer, we use a two-dimensional framelet transform to describe the local spatial correlation.

$$\mathcal{B}_1(\mathcal{X}) = \mathcal{X} \times_1 \mathbf{W}_1 \times_2 \mathbf{W}_2,$$

where  $\mathbf{W}_k$  ( $k = 1, 2$ ) are the framelet transform matrix.

In the second layer, we use a Fourier transform to characterize the global temporal/spectral correlation.

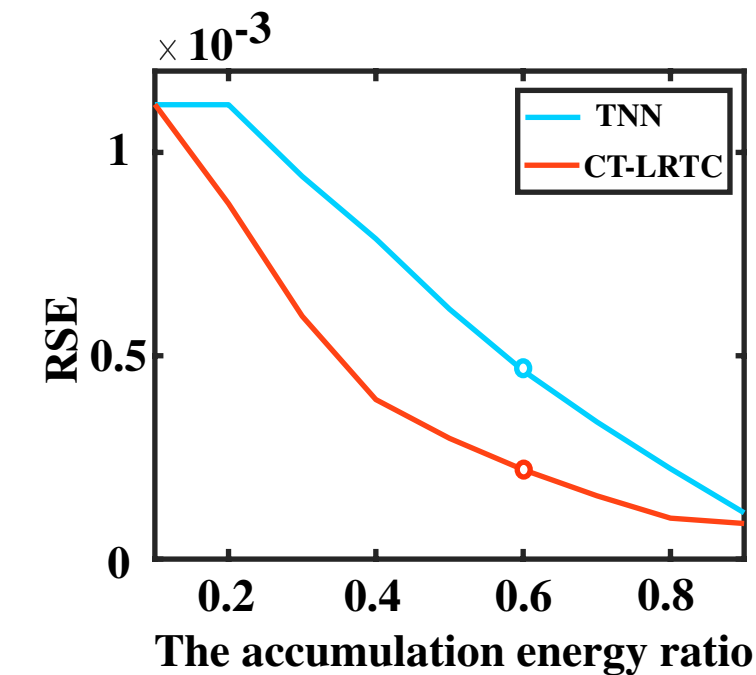
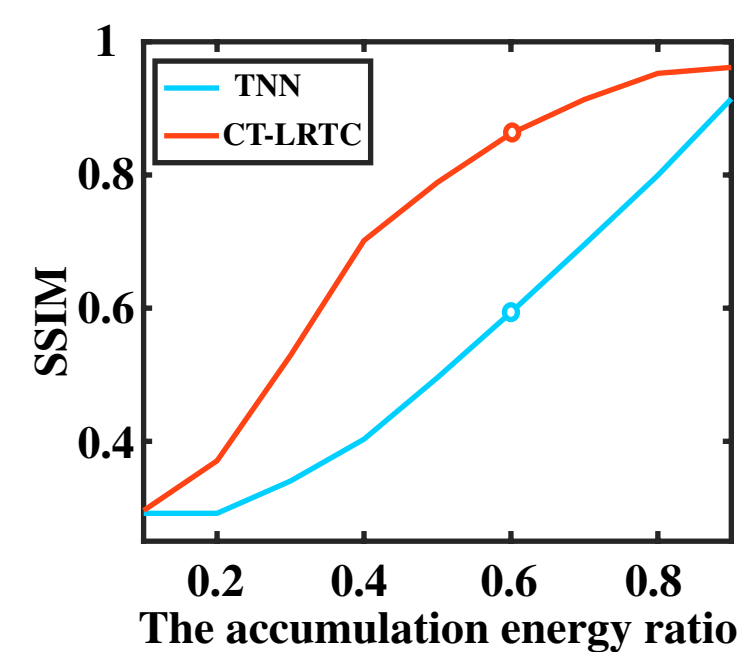
$$\mathcal{B}_2(\mathcal{B}_1(\mathcal{X})) = \mathcal{X} \times_1 \mathbf{W}_1 \times_2 \mathbf{W}_2 \times_3 \mathbf{F},$$

where  $\mathbf{F}$  is a Fourier transform matrix.

In the third layer, we use a Karhunen-Loeve transform (via SVD) to characterize the global spatial correlation.

$$\mathcal{B}_3(\mathcal{B}_2(\mathcal{B}_1(\mathcal{X}))) = \mathbf{KL}(\mathcal{X} \times_1 \mathbf{W}_1 \times_2 \mathbf{W}_2 \times_3 \mathbf{F}),$$

where  $\mathbf{KL}(\cdot)$  is the sum of singular values of each frontal slice of  $\cdot$ .



## Coupled Transforms-Based TC Model

Given a partial observation  $\mathcal{O}$  of the underlying tensor  $\mathcal{X}$ , the coupled transform-based tensor completion (CT-LRTC) model is:

$$\arg \min_{\mathcal{X}} \sum_{k=1}^{n_3} \left\| (\mathcal{X} \times_1 \mathbf{W}_1 \times_2 \mathbf{W}_2 \times_3 \mathbf{F})^{(k)} \right\|_*$$

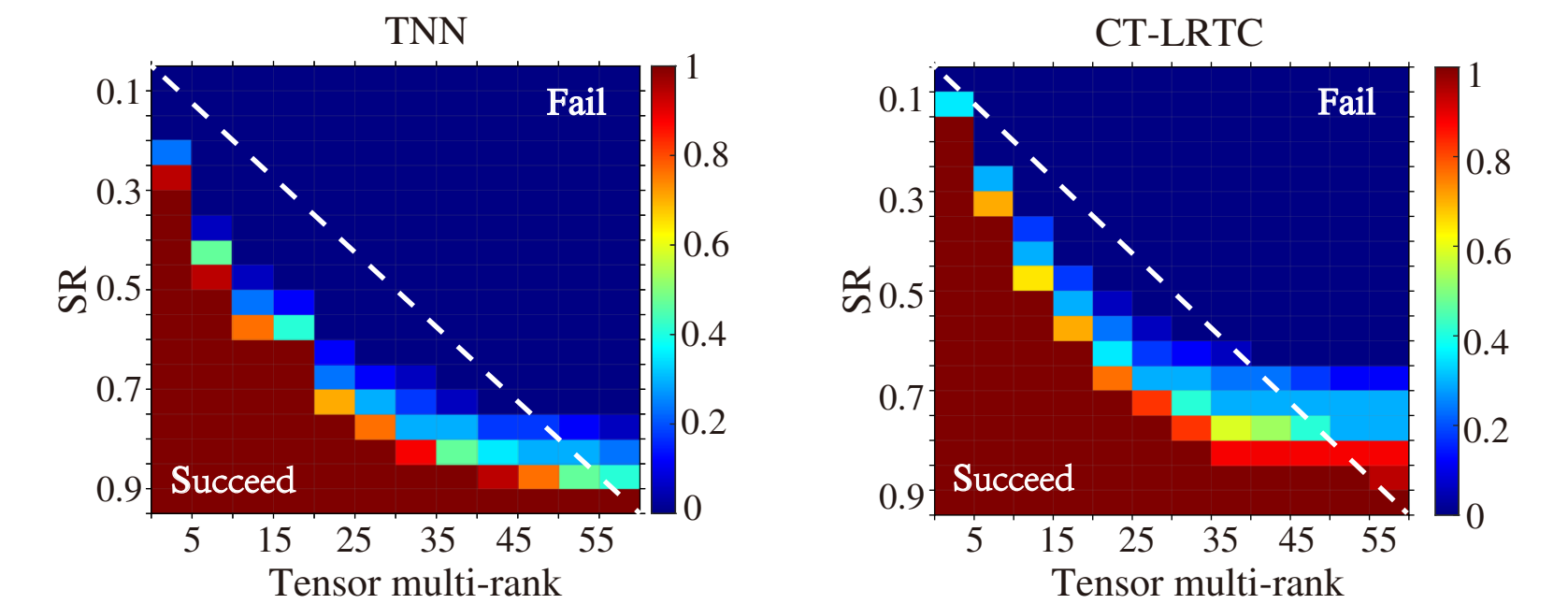
$$\text{s.t. } \mathcal{P}_{\Omega}(\mathcal{X}) = \mathcal{P}_{\Omega}(\mathcal{O}).$$

## Experimental Results

SR: sampling rate

PSNR: peak signal-to-noise ratio

Synthetic Data Experiments (Success Rate)



Multispectral Data Experiments (PSNR)

Dataset	SR	5%	10%	20%	Mean time (m)	Dataset	SR	5%	10%	20%	Mean time (m)
Clay	TNN	39.20	43.53	48.34	8.18	Balloons	TNN	34.99	39.61	44.84	10.43
	TNN-DCT	40.36	44.29	49.09	4.93		TNN-DCT	36.05	40.87	46.16	5.59
	t-TNN	37.80	42.58	48.73	5.12		t-TNN	38.37	43.21	48.30	4.79
	PSTNN	40.19	43.97	48.77	32.33		PSTNN	36.16	40.94	45.63	37.84
	TRLRF	41.86	43.75	47.79	58.78		TRLRF	36.43	40.13	42.35	60.84
	CT-LRTC	44.71	48.16	51.92	68.49		CT-LRTC	41.47	45.07	48.99	100.01

Color Image Inpainting (Visual)

